Scale Dependence of Polarized DIS Asymmetries*

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(1428) Bs.As.
Argentina

18 May 1995

Abstract

We compare the Q^2 dependence of the polarized deep inelastic scattering proton asymmetry, driven by the leading order Altarelli Parisi evolution equations, to those arising from fixed order α_s and α_s^2 approximations. It is shown that the evolution effects associated with gluons, which are not properly taken into account by the leading order approximation, cannot be neglected in the analysis of the most recent experimental data.

*Partially supported by CONICET-Argentina.

Introduction:

Within the last few years, several analysis have been made on the scale dependence of the polarized deep inelastic scattering structure function $g_1(x, Q^2)$, and the related asymmetry $A_1(x, Q^2)$ [1, 2, 3, 4], which has been measured by different experimental groups [5, 6, 7, 8]. Accurate estimates of the magnitude of the scaling violations in these quantities are essential ingredients in the understanding and interpretation of the experimental data. These data are taken at different values of the scale and then used to determine other quantities defined at a common scale, such as the moment of the structure function or parametrizations of parton distributions. The increase in the precision of the experimental data, and the extension of the kinematical range attained in the latest measurements [7, 8], calls for a careful discussion of the different approximations implemented in order to estimate the scale dependence of the data and its consequences in the interpretation of the experiments.

From the theoretical point of view, the main obstacle in the study of the scale dependence comes from the combination of two factors: while the gluon contribution to the structure function, which may be large and essential in the partonic interpretation of the experiments, enters at next-to-leading order (NLO) of perturbative QCD, the corresponding evolution equations have not been calculated yet. In face of this, in most of the attempts, the strategy consists in using the well known leading order (LO) Aaltarelli Parisi (AP) evolution kernels, with quark and gluon distributions defined either at NLO [1], or LO but with an ad hoc gluonic term [2, 3, 4].

While there exists a complete freedom in the choice for the definition of the parton distributions, provided the choice is implemented consistently in other processes, it is clearly inconsistent to evolve them with evolution equations obtained in other schemes of definition. Moreover, a large gluon contribution, as suggested in many analysis of the experimental data [9, 3], may have a crucial role in the evolution of the structure function and it is not clear a priori wherever the AP LO kernels, calculated at an order where there is no gluon contribution to the structure function, will properly account for its role.

Fortunately, there exists an alternative to the usual AP evolution method which may bypass the obstacle mentioned previously. This alternative is based in what is called fixed order perturbation theory and was presented in references [10, 11] in connection with the problem of the evolution of $g_1(x, Q^2)$. Within this method it is possible to write down the structure function in terms of parton distributions at order α_s or α_s^2 and evolve them consistently, exploiting a convenient choice of the factorization scale, which shifts the scale dependence from the parton distributions to already known coefficients. Both fixed order calculations approximate the AP results at LO and NLO, resumming one or two powers of $log(Q^2/M^2)$ respectively.

In this paper we implement the above mentioned method in the analysis of the polarized asymmetries using well defined sets of parton distributions. One

of them with a large gluon density and another without it. Both sets are designed in order to reproduce the asymmetry within the present experimental errors. First, we verify that the fixed order α_s calculation approximates the LO AP results in a wide range of the scale. Then we show that the fixed order α_s^2 evolution, almost equivalent to the NLO AP result, differs from the available AP calculations for the set with a large gluon component and discuss the reason. Finally, we calculate the effects of the correct evolution in the data and the moments of the structure functions and compare these results with those obtained via AP like approximations.

Definitions

In order to unambiguosly define what we mean by fixed order perturbation at a given order, LO and NLO evolution, and our specific choice for the definition of parton distributions, we begin by writing in equation (1) the general expression for the structure function $g_1(x,Q^2)$ in terms of parton distributions, as given in the improved parton model [10]

$$\begin{aligned} g_1(x,Q^2) \quad = \quad & \frac{1}{2}\int_x^1 \frac{dz}{z}\left[\frac{1}{n_f}\sum_{k=1}^{n_f} e_k^2 \left\{\Delta q^S\left(\frac{x}{z},M^2\right) C_q^S\left(z,\frac{Q^2}{M^2}\right) + \right.\right. \qquad (1) \\ & \left.\left. \Delta g\left(\frac{x}{z},M^2\right) C_g\left(z,\frac{Q^2}{M^2}\right)\right\} + \Delta q^{NS}\left(\frac{x}{z},M^2\right) C_q^{NS}\left(z,\frac{Q^2}{M^2}\right)\right] \end{aligned}$$

Δq^S denotes the singlet combination of the polarized quark (Δq_k) and antiquark ($\Delta\overline{q_k}$) densities of species k,

$$\Delta q^S(z,M^2) = \sum_{i=1}^{n_f}[\Delta q_i(z,M^2) + \Delta\overline{q_i}(z,M^2)] \qquad (2)$$

whereas $\Delta q^{NS}(z,M^2)$ denotes the nonsinglet combination,

$$\Delta q^{NS}(z,M^2) = \sum_{i=1}^{n_f}\left(e_i^2 - \frac{1}{n_f}\sum_{k=1}^{n_f} e_k^2\right)[\Delta q_i(z,M^2) + \Delta\overline{q_i}(z,M^2)] \qquad (3)$$

and Δg, the polarized gluon density. The coefficient functions $C_{q,g}$ that multiply each combination of parton distributions are labeled correspondingly and can be calculated at a given order in α_s once the prescriptions for the regularization of ultraviolet singularities and the factorization of those infrared of collinear origin are adopted. Consequently, parton distributions introduced in this way depend on the order of perturbation, on the ultraviolet regularization, and on the factorization procedure.

At order α_s^0, the singlet and non singlet coefficients reduce to the $\delta(1-z)$ function and the gluon coefficient vanishes. There is no need to specify any

prescription due to the absence of singularities. At order α_s^1, there are two classes of contributions to the coefficient, one whose dependence in Q^2/M^2 is logarithmic and another which is not. At order α_s^2 the contributions are classified according to they have $\alpha_s^2 log^2(Q^2/M^2)$, $\alpha_s^2 log(Q^2/M^2)$, $\alpha_s log(Q^2/M^2)$ or non logarithmic dependence. Both the α_s^2 and the α_s have been calculated in references [10] for different factorization prescriptions. There has been a long debate about the way in which the factorization of collinear contributions can be made [12]. We adopt the procedure described in reference [13] which leaves opened the possibility of a non vanishing gluon contribution to the structure function.

The scale M in equation (1), often called factorization scale, is a relic of the factorization procedure and indicates the scale that separates partonic from hadronic effects in the definition. This scale, which in principle would be arbitrary, i.e. provided the coefficient functions were calculated up to infinite order, allows two strategies for the study of the scale dependence of the structure functions. The most used consists in redefining parton distributions in such a way they absorb the scale dependence of the coefficients, and then choosing $Q^2 = M^2$. In this way the dependence on the physical scale Q^2 is shifted from the coefficients to the parton distributions. Then one can use the scale analogously to the renormalization scale, in the procedure that removes the ultraviolet divergences leading to the running coupling constant. A similar procedure in this case leads to the familiar AP equations [14].

What is called LO evolution amounts to calculate up to order α_s the coefficient functions in equation (1) and absorb only the $\alpha_s log Q^2$ term in the redefinition of the parton distributions, thus neglecting the non logarithmic terms. As the gluonic contribution to the structure function enters through non logarithmic terms, it is not present in this approach. The use of the renormalization group techniques whithin the AP evolution equations makes them take into account effectively not only contributions like $\alpha_s log Q^2$, characteristic of the order α_s diagrams, but the whole series of powers of $\alpha_s log Q^2$. The NLO approximation, in turn, takes into account contributions like $\alpha_s^2 log Q^2$ and absorbes all the α_s terms in the redefinition of parton distributions. Then, gluons contribute to the structure functions at NLO, however the coefficients that drive their scale dependence have not been calculated yet.

The second strategy amounts to keep M^2 fixed and let the Q^2 evolution of the structure function proceed via the $log(Q^2/M^2)$ terms in the coefficients. This is called fixed order perturbation theory and leads to the same results of the first method provided $\alpha_s log(Q^2/M^2) << 1$. Satisfied this condition, the leading order evolution is approximated by the fixed order α_s method, as those higher powers of $\alpha_s log(Q^2/M^2)$, present only in AP LO, are not significant. By the same reason, NLO is almost equivalent to fixed order α_s^2; in both approaches terms like $\alpha_s log(Q^2/M^2)$, $\alpha_s^2 log^2(Q^2/M^2)$ and $\alpha_s^2 log(Q^2/M^2)$ are taken into account, the difference being in terms like $\alpha_s^3 log^3(Q^2/M^2)$, $\alpha_s^3 log^2 Q^2/M^2)$ and higher. The same argument applies for the unpolarized structure functions and

in that case this has been verified [11]. As the NLO evolution is not feasible in polarized case, it seems sensible to use fixed order α_s^2 evolution.

Polarized parton distributions

In order to evaluate the actual scale dependence of the structure functions, and through them of the asymmetries, in both evolution strategies one needs a set of parton distributions, at a given scale and defined in accordance to the coefficients or kernels to be used. The problem one confronts then is that the data points coming from the available experiments are given for different values of x and Q^2. In the unpolarized case [15] the problem is solved assuming certain functional x-dependence for the parton distributions and adjusting the parameters in the functional form in such a way the evolved distributions reproduce the data in an iterative process. The number of points to be fitted is more than 30 times greater than that of the paramenters. In the polarized case however, not only the NLO evolution kernels are missing, but the number of points is much more reduced. One has then to look for additional constraints on the parton distributions and perform the analysis at LO. In reference [2] this approach has been followed with a slight variation, which is addmitedly ambiguous, in order to include the gluon contribution to the structure function. In previous analysis to that, a coarser approximation was implemented assuming the asymmetry to be essentially independent of the scale (for a comprehensive review see reference [3]).

In the following we show explicitly the first steps of the iterative procedure. This will allow us to estimate the size of the corrections the experimental data for the asymmetries get when reduced to a common Q^2 value.

The first step in our program consists in obtaining a set of parton distributions that, at some average energy scale, lead to asymmetry values compatible with those obtained experimentaly. The asymmetry is taken to be given by

$$A_1(x,Q^2) \simeq \frac{g_1(x,Q^2)}{F_1(x,Q^2)} \tag{4}$$

where $g_1(x,Q^2)$ is that of equation (1) using coefficients calculated up to order α_s^2 with the factorization prescription already specified. Both M^2 and Q^2 are set to be equal to $10\,GeV^2$, which is an average value of the scales studied by the experiments and will allow us to go to different values of Q^2 keeping $\alpha_s log(Q^2/M^2)$ small enough. The structure function $F_1(x,Q^2)$ is taken to be given by its α_s^2 expression, fed with the very recent set of unpolarized parton distributions of reference [15]. The result can be seen in figures 1 and 2 for different sets of data and for two sets of parton distributions, with a large amount of gluons ($\Delta g = 2.5\,\Delta s = 0$) in set 1 and without gluons ($\Delta g = 0, \Delta s =$) in set 2. Both sets satisfy also other constraints such as asymptotic behaviour and positivity of parton distributions and those coming from hyperon β decays [13]

The second step estimates the error introduced by the assumption about an almost scale independent asymmetry, comparing each data point with the evolved asymmetry. As it can be appreciated in figures 3 and 4, whereas the set without gluons induces a small evolution effect, the asymmetry calculated with parton distributions with a large gluon component exhibits a significant scale dependence. Although the evolution is not negligible, as we were forced to assume in the first step, the actual Q^2 corrections calculated with set 1 (with gluons), rather than invalidate our set, improves the quality of the fit. This improvement is particularly conspicuous when comparing SMC [7] and E-143 [8] low x data to the asymmetry values calculated with the set.

Figures 5 an 6 show the asymmetry calculated at $10\,GeV^2$ compared to the E-143 and SMC data taken at different values of Q^2 and rescaled to $10\,GeV^2$. We rescale the experimental data calculating for each value of x, the difference between the asymmetries at the measured scale and at $10\,GeV^2$, both calculated using our sets as input.

The third and subsecuent steps, modify the original (previous) parametrization in order to improve the quality of the fit. As the precision attained here is more than we need to illustrate our discussion, we stop here for the moment, and use our two sets to compare the different evolution approaches.

Fixed order α_s and α_s^2 evolution

In this section we compare the evolution of the proton asymmetry as given by the α_s, α_s^2 and LO approximations. In figure 7 we plot the result of evolving the asymmetry from $10\,GeV^2$ (solid line) to $3\,GeV^2$ using the three methods and set 1 (α_s: long dashes, α_s^2: short dashes, LO: dots). For small values of x there is a clear cut difference between the α_s^2 and the other approximations. The origin of this discrepancy can be associated to the contributions which are proportional proportional to $\alpha_s^2 log(Q^2/M^2)$ and to the gluon distribution. Both of them are present in the α_s^2 approximation but not in the others. In this region, the LO approximation produces similar results to those of the α_s. For values of x greater than between 0.1, LO is better approximated by the α_s^2, presumably due to the importance of greater terms like $\alpha_s^2 log^2(Q^2/M^2)$, which are included in the latter approximations. The same asymmetry, but calculated with set 2, do not show such a strong dependence on the evolution method (figure 8).

The difference in the Q^2-dependence estimated by the approximations we are analyzing, have non negligible consequences in the moment of the spin dependent structure function g_1, which is calculated from the asymmetry measurement. This can be seen in figure 9, where we show the moment as a function of the lower integration limit. This is computed rescaling the data with the α_s^2 and α_s approximations. The extrapolations were estimated integrating our sets of parton distributions in the unmeasured range. The results are also given in Table 1.

Notice that the difference between the moments comes mainly from the low

x region, where the LO and the α_s approaches give similar results, differing with the α_s^2 estimate. In average the effect of the α_s^2 evolution is to reduce the value estimated with both sets, as it was found in previous LO analysis, but with a more significative correction. The theoretical uncertainty introduced by the evolution is considerably greater than previous analysis, as can be seen in Table 1.

Even though in average the correction to the moment is negative, for each experiment the corrections show differents patterns. For example, analysing E143 data we find them to be positive. This is due to the fact that these data points lay in the region of x where the asymmetries grows with Q^2, and all of them have Q^2 values lower than $10\,GeV^2$. EMC data points belong to the same region, however the evolution to $10\,GeV^2$ is in the opposite direction thus giving negative corrections. For SMC there is an additional negative contribution coming from the low x and low Q^2 data, where the asymmetry decreases with the scale. This emphasizes the importance of taking into account the actual x dependence of the scaling violations, which is not always done.

Conclusions:

The main conclusion of our analysis is that there exists a significant difference between the α_s and α_s^2 evolution of the asymmetries at small values of the variable x. This means that a non negligible difference will be found between the LO and NLO analysis. As the small x data, which is usually taken at low values of the Q^2-scale, are crucial in the estimation of the moments, these corrections must be taken into account. The analysis beyond leading order emphasizes the difference between the evolution at small and large x, which is essential when different sets of data are compared. Given that the fixed order technique is consistent and more accurate than the ambiguous mixture of perturbation orders in the other approaches, it seems sensible to use it in forthcoming analysis of polarized deep inelastic scattering data.

Acknowledgements:

One of us (CAGC) acknowledges the Centro Brasileiro de Pesquisas Fisicas (CBPF), Rio de Janeiro, Brazil for the warm hospitality extended to him during a recent visit in the framework of the TWAS Associate Membership Scheme at Centres of Excellence.

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Figure captions

Figure 1 Polarized deep inelastic scattering asymmetry data coming from SMC and E-143 experiments and calculated with set 1 and set 2 at $Q^2 = 10GeV^2$.
Figure 2 Polarized deep inelastic scattering asymmetry data coming from E-130, E-80 and EMC experiments and calculated with set 1 and set 2 at $Q^2 = 10GeV^2$.
Figure 3 Q^2 dependence of the asymmetry (set 1, $\mathcal{O}(\alpha_s^2)$)
Figure 4 Q^2 dependence of the asymmetry (set 2, $\mathcal{O}(\alpha_s^2)$)
Figure 5 The asymmetry at $Q^2 = 10GeV^2$ and the SMC data rescaled to that value.
Figure 6 The asymmetry at $Q^2 = 10GeV^2$ and the E-143 data rescaled to that value.
Figure 7 Evolution effects on the asymmetry coming from the different methods (set 1).
Figure 8 Evolution effects on the asymmetry coming from the different methods (set 2).
Figure 9 Evolution effects on the moments of g_1^p coming from different methods and parton distributions.

Table caption

Table 1 Moments of g_1^p estimated assuming no scale dependence in the asymmetry (naive), and α_s^2 evolution (the unmeasured interval calculated with the sets).

Exp.	Naive	Set 1 α_s^2	Set 2 α_s^2
EMC	0.128	0.120	0.126
E143	0.126	0.139	0.129
SMC	0.134	0.113	0.130

Table 1.

This figure "fig1-5.png" is available in "png" format from:

http://arXiv.org/ps/hep-ph/9505377v1